LIBERTARIANISM AND SOCIAL CHOICE:
AN APPRAISAL OF ECONOMIC THEORY

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This paper discusses the seminal theoretical frameworks provided in Welfare Economics to incorporate individual liberty in social choice making; in particular, the nature and the cause of the problems in incorporating the ideas of the Nobel laureate Sen about libertarianism in social choice making; and whether the alternative frameworks provided by Sen's critics help resolve those problems.

I. INTRODUCTION

According to J.S.Mill(1848), "-----there is a circle around every individual which no government, be it that of one, of a few, or of the many, ought to be permitted to overstep". All economists agree that each individual should have a Recognised Personal Sphere (RPS) with which the rest of the society should not be allowed to interfere.

Sen(1970) was the first economist to introduce a formal framework to incorporate this personal sphere of the individual in social choice making. According to him, every individual should have some minimal liberty in choice making; given other things in the society, if one prefers pink walls to white then society should allow one to have this, even if a majority of the community would like to see one's walls white. Similarly whether one sleeps on one's belly or one's back is a matter in which the society should permit one absolute freedom, even if a majority of the community is nosey enough to feel that one must sleep on one's back.

But the inclusion of these ideas of libertarianism in the societal choices leads to several problems recognised by Sen himself and his critics. Another framework, on the same lines as Sen, was provided by Gibbard(1974) claiming to overcome the problems that Sen's framework came across. But the critics of Sen (Gaertner et al.,1992), contend that Sen's formulation does not capture the intuitive notion of individual liberty since it looks at the individual preferences over social states rather than over particular aspects of the social state over which the individual's decision matters as regards social choice. So Gaertner et al. provide an alternative framework using game forms which looks at individual choices over particular aspects of the alternatives rather than social states as a whole.

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It is in this backdrop that this paper is organised as follows: Section II looks at the definitions and basic notations; Section III provides Sen's formulation of individual rights and the problems that arise in including them in social choice making; Section IV looks at Gibbard's formulation; Section V provides an alternative framework using game forms; Section VI gives an overview of the debate between Sen and his critics and looks at whether game form formulation can provide an escape route to the problems that we face in Sen-Gibbard formulations; and, finally, Section VII provides the concluding points.

II. BASIC NOTATIONS AND DEFINITIONS

N stands for the society consisting of n individuals-1,2, _ _ _ _,n

(Individuals are denoted by i,j,_ _ _ _).

X denotes the set of all social alternatives [X={x, y,_ _ _ _ _ _ }].

$\chi \rightarrow$ denotes the set of all non empty subsets of X.

A is the set of available alternatives.

$\Re$ denotes the set of all orderings[1].

$\Re_a$ stands for the set of preference relations which are reflexive, complete and acyclic[2].

R_i is the ordering of the ith individual in the society and P_i and I_i are respectively the asymmetric and symenetric factors.

$R \in \Re$ denotes the social preference relation obtained by some rule of aggregation of individual preferences.

The n-tuple of preference orderings $(R_1$ _ _ $R_n)$ is called a preference profile and is denoted by s. It is clear that s $\in \Re^n$.

Definition 1: *Social Welfare Function* (SWF): A Social Welfare Function is a function f such that f: $\Re^n \rightarrow \Re$ such that $\forall (R_1, \_ \_ \_ \_ \_, R_n) \in \Re^n$, R = $f(R_1, \_ \_ \_ \_ \_, R_n)$. In words, an SWF is a function that specifies one and only one social preference ordering R for each set of N individual orderings.

Definition 2: *Social Decision Function* (SDF): A Social Decision Function is a function g: $\Re^n \rightarrow \Re_a$ such that $\forall (R_1, \_ \_ \_, R_n) \in \Re^n$, R = g $(R_1, \_ \_ \_ \_, R_n)$.

The SDF and SWF are aggregation rules which aggregate individual preference orderings to a social preference relation. While the SWF aggregates a preference profile to derive a social preference ordering, the SDF aggregates a preference profile to derive a social preference relation satisfying reflexivity, completeness and acyclicity. It is clear that an SWF implies an SDF but the converse is not true.

Definition 3: *Social Choice Function* (SCF): An SCF is a function C: $\chi * \Re^n \rightarrow \chi$, such that $\forall$ A $\in \chi$, & all s $\in \Re^n$, C(A; s) $\subseteq$ A.

C(A; s) is to be interpreted as the set of socially chosen alternatives from the available set when s is the profile of preference orderings of the individuals in the society.

Since the chosen set is a subset of A which in turn is an element of χ, the choice set can never be empty.

Definition 4: *Rights System* (RS): An RS is an assignment of ordered pair of alternatives to individuals. It will be a set of ordered triples of the from <x,y,i> i.e. the alternative (x,y) being assigned to individual i by the RS.

Definition 5: *Rights Implementation Rule*(RIR): A right to x over y is implemented by C if $\forall$ A $\in$ χ, $\forall$ x,y $\in$ A, $xP_iy \rightarrow y \notin C(A;R)$.

Definition 6: *Weak Pareto* (WP): $\forall x$, y $\in$ X, if xP_iy $\forall i$ $\in$ N then xPy .

Which means that if every individual prefers x to y then society must prefer x to y. It is considered to be a weak welfaristic requirement i.e. a weak requirement of Welfarism (which claims that individual utilities are the sole basis of individual choices).

III. SEN ON INDIVIDUAL RIGHTS

The original framework of Sen was in terms of preferences but we will look at its conterpart in terms of choices so as to have consistency in the analysis.

Sen's Libertarianism Condition (SL): $\forall i \in$ N, $\exists$ distinct x,y $\in$ X such that [if

x $\in$ A & xP_iy then y $\notin C(A;s)$] and [if y $\in$ A& yP_ix then x $\notin$ C(A;s)] i.e. for every individual i there is at least one pair of alternatives, say (x,y) such that <x,y,i>,<y,x,i> $\in$ RS which is implemented by C. The intention is to permit each individual the freedom to determine atleast one social choice.

Sen's Minimal Libertarianism Condition (ML): $\exists$ distinct i,k, $\in$ N & x,y, z, w $\in$ X where $x \neq y$ & $z \neq w$, such that [if x $\in$ A & xP_iy then y $\notin C(A;s)$]; [if y $\in$ A& yP_ix then x $\notin C(A;s)$] and   [if z $\in$ A& zP_kw then w $\notin C(A;s)$]; [if w $\in$ A& wP_kz then z $\notin C(A;s)$] ,i.e.there are atleast two individuals such that for each of them there is atleast one pair of alternatives over which they are decisive(see Sen, 1992).

Liberal Paradox

There exists a conflict between the libertarianism conditions laid by Sen and WP, i.e. it has been found that there does not exist any SCF which can simultaneously satisfy WP and SL. Even if SL is weakened to ML, the impossibility stays.

Theorem 1: There is no social choice function that can simultaneously satisfy WP and ML (Since SL is stronger than ML the same proof holds for a conflict between WP and SL).

Proof: let there be two individuals 1 and 2 and the two pairs of alternatives

referred to in ML be (x,y) and (z,w), respectively.

There can be three possibilities :

(1) (x,y) and (z,w) are identical. There arises a contradiction in this case since if both the individuals are decisive over the same set of alternatives then with counter preferences(i.e. with individuals 1 and 2 and preferences xP_1y and yP_2x, C(x,y) would be null) the choice set would be null leading to a contradiction.

(2) (x,y) and (z,w) have one element in common, say x=z. Assume now that 1 prefers x to y and 2 prefers w to z(=x). And let everyone in the society prefer y to w .

We have the following preferences :

1: $xP_1y\ P_1w$

2: $yP_2wP_2z(=x)$

$\forall i \in N-\{1,2\}$: $y\ P_iw$.

If A={x,y,w}

By 1's rights, $y \notin C(A,s)$

By 2's rights, $x(=z) \notin C(A,s)$

By WP, $w \notin C(A,s)$

Therefore, the choice set would be null in this case which contradicts our definition of an SCF.

(3) Let x,y,z and w be all distinct
Let the preferences be as follows

1: $wP_1x\ P_1yP_1z$

2: $yP_2zP_2wP_2x$

$\forall i \in N-\{1,2\}$: $y\ P_iz\ wP_ix$.

Let A={x,y,z,w}

By 1's rights, $y \notin C(A,s)$

By 2's rights, $w \notin C(A,s)$

By WP, z, $x \notin C(A,s)$

Therefore, C(A,s) would be a null set given the above preferences which leads to a contradiction of our definition of choice function.

Thus, there is no SCF satisfying conditions WP and ML (SL).This impossiblity is known as the Liberal Paradox.

Let us look at an example to show the above result:

Suppose there are two individuals in the society (1 and 2). There is one copy of the book *Lady Chatterly's Lover*. There are hence three alternatives:

(i) Individual 1 reads it (x).

(ii) Individual 2 reads it (y).

(iii) No one reads it (z)

Person 1, Prude, prefers most that no one reads it but given a choice between either of the two reading it, he would prefer to read it . So his preferences are: $zP_1x\ P_1y$.

Person 2, Lude, prefers that either of them reads it rather than the book going waste. But given a choice, he takes more delight in the thought that 1 reads it rather than he reading it himself. His preferences are hence: xP_2yP_2z.

Since both the individuals have x P_iy, by WP, the society should have xPy.

Let $A=\{x,y,z\}$

Since the choice over (x,z) comes in 1's RPS, by Sen's libertarianism condition, his choice over the pair should matter and hence $x \notin C(A,s)$.

Similarly, 2 should be decisive over (y,z), and so $z \notin C(A,s)$.

By WP, $y \notin C(A,s)$.

Hence, $C(\{x,y,z\})$, would be a null set, leading to a contradiction of the definition of the SCF.

IV. GIBBARD ON INDIVIDUAL RIGHTS

In his framework, Gibbard (1974) assumes that each social state is decomposable into certain issues which further have a finite set of feature alternatives. Suppose there are m issues in each social state , for each issue $k=1,2\_\_\_\_,m$, let there be a finite set T_k of feature alternatives, each with atleast two elements.

Assuming that the features are technologically separable from each other, the set T of all technologically possible alternatives is the cartesian product $T_1{}^*T_2{}^*\_\_\_\_\_{}^*T_M$. $x \in T$ implies that $x_1 \in T_1, x_2 \in T_2, x_3 \in T_3,\_$ $\_\_\_\_, x_m \in T_m.$ For example, suppose we want to determine the wall colour of individual 1,which is one of the issues of the social alternative. The possible feature alternatives' set can be {blue, white,_____}.

Definition 7: j-variants: $x, y \in X$ are j-variants if $\forall k, k \neq j$, $x_k = y_k$

Gibbard's Libertarianism Condition
(GL) : $\forall i \in N$, $\exists$ one private issue j such that the RS assigns all j-variants to i and is implemented by C.

Gibbard's formulation GL, though seems to be the same as ML in granting individuals the right over social states, is actually quite distinct from ML since GL gives to i the decisiveness over-all pairs of j-variants whereas under ML the individual may not have such decisiveness over more than one pair of distinct alternatives.

Paradoxes with Gibbard's Formulation

Firstly, note that the decomposability of the alternatives is a very strong assumption and it may not be possible to analyse certain types of problems in a framework where the alternatives are assumed to be decomposable. Consider the following example:

There are 2 individuals in the society 1 and 2. 1 has the right to invite or not to invite 2 to his/her house. If 1 invites 2, then 2 has the right to go or not to go to 1's house.

Thus, there are three possible outcomes:

(i)   1 invites 2 and 2 goes to 1's house

(ii)   1 invites 2 and 2 does not go to 1's house

(iii) 1 does not invite 2 (and 2 does not go to 1's house)

(Note that the alternative that 1 does not invite 2 and 2 goes to 1's house is ruled out).

In this case, the set X of social alternatives cannot be represented as the cartesian product of x and y where x={1 invites 2, 1 does not invite 2} and y={2 goes to 1's house, 2 does not go to 1's house}.

The main problem arises out of interdependence between the two types of decisions. However, the assumption of decomposition does provide a convenient framework for many cases where such interdependence may not exist.

With the Gibbard formulation, one does not even need to bring in WP criterion; there arises a self inconsistency in that case.

Consider two individuals who have to choose their wall colour. 1 is a perverse non-conformist: wants his wall colour different from 2's and 2 is a perverse conformist: wants his wall colour to be the same as 1's.

If two wall colours are available: blue(b) and white(w), everything else remaining the same, four possibilities can arise :

a_{bb}: wall colour for both is blue

a_{wb}: wall colour for 1 is white and for 2 is blue

a_{bw}: wall colour for 1 is blue and for 2 is white

a_{ww}: wall colour for both is white

According to GL, 1 is decisive over all 1-variants and 2 is decisive over all 2-variants.

Preferences of the two individuals can be described as

$1 : a_{wb} \, P_1 \, a_{bb} \rightarrow$ (by 1's rights) $a_{bb} \notin C(A,s)$

$a_{bw} \, P_1 \, a_{ww} \rightarrow$ (by 1's rights) $a_{ww} \notin C(A,s)$

$2 : a_{ww} \, P_2 \, a_{wb} \rightarrow$ (by 2's rights) $a_{wb} \notin C(A,s)$

$a_{bb} \, P_2 \, a_{bw} \rightarrow$ (by 2's rights) $a_{bw} \notin C(A,s)$

If $A=\{ a_{bb}, a_{wb}, a_{bw}, a_{ww}\}$

C(A,s) would be a null set leading to a contradiction.

Theorem 2: No SCF with N>1 satisfies GL.

Proof : Assume that there are 2 individuals 1 and 2 and there exists a SCF satisfying condition GL. Assume that 1 is decisive over all 1-variants and 2 is decisive over all 2-variants.

Suppose there are four alternatives

$x=(a_1, a_2, a_3, _____, a_m)$

$y=(a_1, b_2, a_3, _____, a_m)$

$z=(b_1, b_2, a_3, _____, a_m)$

$w=(b_1, a_2, a_3, _____, a_m)$

1 is decisive over (x,w) and (z,y)

2 is decisive over (x,y) and (z,w)

Consider the following preference profile:

1: $yP_1wP_1xP_1z$

2: $zP_2xP_2wP_2y$

Let A={x,y,z,w}
By 1's rights z, $x \notin C(A,s)$
By 2's rights y,w $\notin C(A,s)$.

Therefore, C(A,s) would be a null set leading to a contradiction.

According to Gibbard, this conflict arises since individual preferences over social states rather than a particular feature is taken into account. There is hence a conditional preference eg. In the above proof the libertarian claim states that if 1 want my wall colour to differ from 2's wall colour then it is better to have my walls white and 2's blue rather than having the walls of both as blue.

A reasonable libertarian claim would then disregard such conditional preferences. One would have an unconditional preference for white walls over blue if other things being equal, he prefers white walls to blue regardless of any other feature of the world.

Definition 8: *Unconditional Preferences* : Let x and y be feature alternatives in X_1, then for a given P, i prefers x to y unconditionally ($xP_i^u y$) for issue j iff (if and only if)for every pair of j-variants x and y , if $x_j = x$ and $y_j = y$ then $xP_i y$.

Condition GL' : For each individual i there is an issue j such that for every pair of j-variants x and y whenever $xP_i^u y$ [i prefers x_j to y_j unconditionally for issue j] then xPy. There exists an RS' which incorporates these preferences and is implemented by C.

Theorem 3: No SCF with N>1 satisfies GL' and WP.

Proof : Suppose there is an SCF satisfying GL' and WP. Since N>1, suppose N=2.

Individual 1 is decisive over all 1-variants and 2 is decisive over all 2-variants.

Let

$x=(a_1, a_2, a_3, \_ \_ \_ \_ \_ \_, a_m)$
$y=(a_1, b_2, a_3, \_ \_ \_ \_ \_ \_, a_m)$
$z=(b_1, b_2, a_3, \_ \_ \_ \_ \_ \_, a_m)$
$w=(b_1, a_2, a_3, \_ \_ \_ \_ \_ \_, a_m)$

Individual 1 is decisive over (x,w) and (z,y)

Individual 2 is decisive over (x,y) and (z,w)

Consider the following preference profile

1: $xP_1 wP_1 yP_1 z$

2: $zP_2 wP_2 yP_2 x$

$\forall i \in N-\{1,2\}: wPy$

According to 1's preferences, 1 has an unconditional preference for a_1

According to 2's preferences, 2 has an unconditional preference for b_2

Let A={x,y,w}
By 1's rights w $\notin C(A,s)$
By 2's rights x $\notin C(A,s)$
By WP, y $\notin C(A,s)$

Therefore, C(A, s) would be a null set leading to a contradiction.

In these cases where the libertarianism condition conflicts with WP, a cycle of preferences is found. Gibbard suggests a modification of the Rights Implementation Rule so as to break this cycle.

Gibbard's Condition for Rights Waiving

For any preference profile s and any available set A, an individual's rights to x over y is waived under RS iff for some z $(z \neq y)$, $yR_i z$ and for some available sequence of alternatives in A, $\Sigma = y^1, y^2, \_ \_ \_ \_ , y\lambda$

(i) $y^1 = z$

(ii) $y\lambda = x$

(iii) For every $\mu = 1, 2 \_ \_ \_ \_ \_, \lambda-1$ atleast one of the following holds:

(a) $\forall j : y^{\mu} P_j y^{\mu+1}$

(b) $\exists j : j \neq i, <y^{\mu}, y^{\mu+1} j> \in$ RS and $y^{\mu} P_j y^{\mu+1}$

If i's rights to x over y are waived then we write $xW_i y$.

The Rights Waiving Principle helps in a situation where one person takes a perverse interest in the affairs of another. Consider the following example:

Angelina wants to marry Edwin but would settle for the judge who wants whatever she wants. Edwin wants to remain single but would rather marry Angelina rather than see her getting married to the judge.

There are then three alternatives:

w_E : Edwin weds Angelina
w_J : Angelina marries the Judge
w_O : Both Edwin and Angelina stay single.

Angelina's preferences are : $w_E P_A w_J P_A w_O$
Edwin's preferences are : $w_O P_E w_E P_E w_J$

By Angelina's Rights, w_O should not be chosen.

By Edwin's Rights, w_E should not be chosen.

Since both Angelina and Edwin prefer w_E to w_J so by WP w_J shouldn't be chosen.

Hence, $C[\{w_E, w_J, w_O\}, s] = \Phi$ leads to a contradiction.

We have the following cycle in this: $w_J P w_O, w_O P w_E$ & $w_E P w_J$.

Breaking Cycle: At What Point?

It could be broken at $w_E P w_J$. w_J is optimal on account that it breaks the cycle and Angelina should marry the judge.

But the individuals can be allowed to bargain here. Angelina has every right to marry the judge but she prefers Edwin ; Edwin has every right not to wed Angelina but if he wants to prevent her from marrying the judge then he must wed her himself. Left freely to bargain away their rights, Edwin and Angelina would agree to the outcome w_E.

Hence, a libertarian might hold that deplorable though Edwin's motives be w_E is a just outcome under the circumstances so the cycle must be broken at the step $w_O P w_E$

That means that we deny Edwin the right to w_O over w_E. He has the right to remain single but the right is alienable(he can bargain it away to keep Angelina from marrying the Judge).

If Edwin exercises his right to avoid w_E, he gets something he likes no better(w_J). In such cicumstances, even though Edwin has a right to w_O over w_E and prefers w_O to w_E, w_E may still be optimal. It may be to Edwin's advantage to waive his right to w_O over w_E in favour of the Pareto principle.

There exists a sequence $\Sigma = w_J w_O$ satisfying the condition of the rights waiving principle. So Edwin's rights will be waived, w_J is Pareto dominated and hence w_E will emerge as the solution.

Condition GL" :For every individual i, there is a j such that for every pair of j-variants x and y, C accords i an alienable right to x over y

Gibbard claims that GL" is consistent with the Pareto principle i.e. condition GL" and WP can be jointly satisfied so long as there are enough features to go around with (i.e. so long as the number of features in an alternative is atleast as great as the number of people in the society)

But is GL" really a libertarian claim? Are alienable rights strong enough to satisfy a libertarian? Having a right to something should give a person a decisive say on the matter so that normally on that issue he can stand against the rest of the world. Unless it is

waived, an alienable right lets a person decide an issue completely. And the main drawback with the Rights Waiving principle is that there is voluntariness in rights waiving i.e. it depends on the individual himself whether he is willing to waive his right over a pair of alternatives.

V. PROBLEMS WITH SEN-GIBBARD AND ALTERNATIVE GAME FORM FORMULATIONS

It has been contended by Gaertner et al. (1992) that Sen's formulation does not capture the intuitive notion of individual rights (the basic concern being the formulation of individual rights *per se* and not the conflict between individual rights and the weak welfaristic requirement of Pareto optimality, the latter conflict being a more fundamental problem in the theory of social choice).

Important aspects of Sen's formulation are:

* It is an individual's preference over two complete descriptions of the society (i.e. over two social states), differing only w.r.t that individual's RPS, which constrains social choice.

* SL was intended to be only a necessary condition for individual I to have a right, it was not meant to be a necessary and sufficient condition which would capture the entire content of the notion of individual rights.

So it would be quite unreasonable if one were to criticize SL on the ground that it failed to capture some part of our intuition about individual rights, however important that part may happen to be.

On the other hand, it would be a shortcoming of formulation SL if it turns out to be inconsistent with some important aspect of the critics' intuition about individual rights.

Here is a counter example to show that SL is inconsistent with the critics' intuitive notion of liberty.

Counter Example: Two individuals: Conformist(1) and non-conformist(2). Each individual has the right to choose his shirt from among the shirt he owns. Each individual has two shirts one white(w) and one blue(b). The two individuals are completely ignorant about the other's preferences and at the time of making his choice each individual is completely ignorant about the other's choice.

The two individual's preference orderings are:

1: $(w,w)P_1(b,b)P_1(b,w)P_1(w,b)$

2: $(b,w)P_2(w,b)P_2(w,w)P_2(b,b)$

1 would like to have w rather than b if 2 has w and b rather than w if 2 has b. 2 would like to have b if one has w and w if 1 has b.

If we want to formulate the right of each individual in Sen's terms then given our intuition about each individual's RPS

atleast one of the conditions (i) and (ii) should be true and one of (iii) and (iv) should be true.

(i)   1 is decisive over $\{(w,w),(b,w)\}$

(ii)   1 is decisive over $\{(w,b),(b,b)\}$

(iii) 2 is decisive over $\{(b,w),(b,b)\}$

(iv) 2 is decisive over $\{(w,b),(w,w)\}$

The critics show that three serious intuitive difficulties (Problems A, B and C) arise in the context of the formulation of SL and some of its natural extensions. Problem A affects SL as well as its extensions where as B and C affect the extensions of SL .

Problem A: Suppose that (i) holds. Let the two individuals freely choose their shirts without knowing anything about the other individual's choice and preferences. Suppose given such ignorance, 1 follows the maximin principle[3] and chooses b. Similarly suppose 2 follows the maximin principle and chooses w. Then (b,w) will emerge as the social outcome. This would, of course, be inconsistent with (i). However, given that (b,w) emerges from the two individual's free choices of their respective shirts, very few people would be willing to say that there was a violation of the right of any individual to choose his shirt.

The fact that each individual is free to choose the colour of his shirt without any external constraint seems to capture the entire intuitive content of the critics' conception of the rights under consi-

deration. Even if as a result of exercising this freedom of choice, SL is violated, it doesn't justify us in saying that there has been a violation of the right of either Individual.

In a similar way, for each of the conditions (ii), (iii) and (iv), it is possible to specify the indiyidual's preference orderings in such a way that, given the maximin behaviour on the part of each individual, the free choice of the individuals would give rise to a social outcome that would contradict the condition under consideration.

Note that the assumption of maximin behaviour under uncertainty is not crucial to the above reasoning. Given any rule of choice under uncertainty, the features of the example can be modified to get the above kind of inconsistency.

Problem B: Problem A arises irrespective of whether 1 is decisive over only one of (i) or (ii) or over each of the two sets. However, given that each of (i) and (ii) differ only as regards the colour of 1's shirt, it is not clear why 1 should be decisive over one of the two two-element sets but not the other. It is more natural to assume that 1 is decisive over both the two-element sets. But such an assumption leads to another kind of intuitive problem i.e. a type of power for Individual 1 if he is decisive over (i) as well as (ii) that is completely inconsistent with our intuition.

Since 1 is decisive over (i) and (ii)

both, by choosing b he can make sure that the outcome never lies in {(w,w),(w,b)} and by choosing w he can make sure that it never lies in {(b,b),(b,w)}.

Problem C: But the question that arises here is that why should we give only 1 the right to both (i) and (ii) and not give the same right to 2 over (iii) and (iv).

Suppose we do so. Then irrespective of the social outcome one of these conditions is bound to be violated which implies the violation of someone's right.

Root of These Problems

Under the critics' conception of the right of an individual to choose the colour of his shirt, the individual enjoys the right to determine a particular aspect or feature of the social alternative; and when he makes his choice w.r.t this particular aspect, his choice imposes restrictions on the final social outcome in so far as, in the final social outcome, that particular aspect must be exactly as he chose it to be.

In contrast, SL doesn't mention individual's ability to determine a particular aspect of the social alternative. Instead the constraint on social choice is linked to the individual's preference over some pairs of the social states or complete descriptions of all aspects of the society.

If each possible choice by an individual w.r.t a particular aspect of the social state, coming within his RPS, was linked

to exactly one social state, then there would be an obvious and tight connection between' such choice by the individual and his preference over the social states or complete descriptions of the society. However, in the above example no such tight connection exists. Therefore, we consider an alternative framework with the required connection.

Game Form

A game form representation of a rights structure is simply a specification of a (normal) game form $<A;S_{1A},_____,S_{1A};G_A>$ where

A is as usual the set of feasible alternatives;

S_{iA} is the strategy of individual i;

G_A is the outcome function which specifies exactly one outcome for each element of the n-tuple of strategies, one strategy for each player (the players being the members of the society).

O is the set of all possible outcomes.

The game form $<A;S_{1A},_____,S_{1A};G_A>$ is intended to capture the powers, freedoms, and claims conferred on the individuals by the·rights structure. But note that it is a game without the specification of individual preferences.

The content of individual rights in this framework lies in a specification of the admissible strategies for each player j ∈ N, and the complete freedom of each individual to choose any of the admi-ssible strategies and/or the

obligation of the agents not to choose a non-admissible strategy.

Let us look at an example:

There are two individuals in the society-1 and 2; 1 is a smoker and 2 is a non-smoker.

The strategy set of each individual is:

$S_1=\{Smoke, do not smoke\}$;

$S_2=\{object if one smokes, do not object\}$

The set of outcomes is: $O=\{x, y, z\}$

where $x=G_A\{1$ smokes,2 objects)=1 stops smoking

$y=G_A\{1$ smokes,2 does not object)=1 continues smoking

$z=G_A\{1$ does not smoke,_ _ _)=1 does not smoke

(Note that the strategy set of 1 does not contain the strategy of continuing smoking despite 2's objection)

This formulation of individual rights via game form formulation accomodates most instances of what we would intuitively think of as individual rights, which cannot be so easily accomodated by formulation SL.

General Remarks on Game Forms

(1) The notion of a game form, by itself, has very little to do with rights. All that it claims is that rights are best modelled as game forms, with strategy sets being interpreted as the sets of legally or socially

admissible strategies for each and every agent.

(2) How does the society decide which strategies should or should not be admissible for a specific player in a given context? In other words how are rights granted? The important aspects of the rights of an individual i in game form formulation always takes the form of i's freedom to choose a strategy from the set of all admissible strategies for him, and/or a specification of the admissible strategies of some other agents.

(3) The game form formulation is in terms of normal game forms. Thus, the intuition of certain extensive games cannot be adequately captured in terms of games in normal form. By relying on the normal game form, the game form formulation also suffers from this problem.

However, despite this, it would seem to provide a far more flexible and adequate framework for analysing a wide range of rights than is provided by the classical formulation in Sen's terms. The main difference between Sen's and the game form approach is that Sen's conception of an individual's right involves restrictions on social choice which are contingent on the nature of individual's preferences over certain pairs of social alternatives. In contrast, the game form formulation of individual rights does not refer at all to individual's preferences

over social alternatives, nor does it refer to the actual outcome of any game. The game form approach claims that, given a feasible set, A, of social alternatives, the substantive content of the rights that individuals enjoy is entirely captured by the sets of permissible strategies of the individuals and the outcome function that specifies the outcome for each possible n-tuple of permissible strategies.

The game form approach does not recognise any conceptual link between an individual having a right and that individual's preferences over different social alternatives and the actual social choice of an alternative.

It is only when we come to the issue of the exercise of rights that the game form approach recognises the relevance of individual preferences over social alternatives: given the powers, freedom, claims and immunities which are guaranteed by the rights structure G , the individual's choice of strategies and hence the actual social outcome, will depend on the individual preferences over the social alternatives.

The question that arises here is whether game forms help resolve the Liberal paradox. Consider an example where there are two individuals who have to choose the colour of their walls. There are two possible colours that they can choose from: blue (b) and white(w). There are four possible outcomes (b,b), (b,w),(w,b) and (w,w).

The preferences of the individuals are:

1: $(b,w)P(w,w)P(b,b)P(w,b)$

2: $(w,b)P(w,w)P(b,b)P(b,w)$

so. that both the individuals have b as their dominant strategy[4] and hence (b,b) would appear as the social outcome which is Pareto dominated by (w,w) so that the Liberal Paradox persists.

Pattanaik(1994;1996) argues that the conflict that arises here is due to the behavioural assumptions underlying individual choices; otherwise there is no direct conflict between Paretianism and liberalism underlying game forms.

But the fact remains that the Liberal Paradox still persists under the game form formulation of individual rights.

VI. DEBATE BETWEEN SEN AND HIS CRITICS

In their extensive analysis of the need for reformulating the social choice characterisation of liberty, Gaertner et al.(1992) do not claim that such reformulation resolves the impossibility result. Indeed, they argue that the problem persists under virtually every plausible concept of individual rights that one can think of.

According to Sen(1992), it would thus be a mistake to identify the substantial arguments that have been presented by Gaertner et al. and Nozick(1974) in favour of the case for reformulating the characterisation of liberty in social choice theory with the more contingent and not particularly engaging claim that

the impossibility problem could be resolved in this way.

The main issues concerning the formulation of social choice theory are:

(1) Necessity, not sufficiency:

According to Sen, there were two distinct aims in introducing the concept of liberty and rights in social choice theory. The more general aim was to work toward an extension of the Classic social-choice format due to Kenneth-Arrow, by making explicit room for rights in general and liberty in particular. The more specific aim was to try to capture in formal terms the tension between considerations of liberty and rights, on the one hand, and the exclusively utility based principles standardly used in welfare economics, on the other. This was done in the form of a simple impossibility result.

Since the motivation behind such an impossibility theorem requires the use of as weak a condition as is adequate for the purpose, there was no attempt in that context to axiomatize anything like the full demands of liberty, but only to identify one of the implications of such demands.

ML tries to get a weak condition that is entailed by different possible fuller formulations of the requirements of liberty, and could thus be seen as a necessary but not a sufficient condition for guaranteeing liberty in a society.

(2) Aspect choice entails outcome choice but not vice-versa:

According to Sen, it is clear that liberty should demand more than what ML does, but it is not so easy to agree on a particular set of elaborated demands. One claim that has sometimes been made is that a person should be able to determine one aspect of any social choice that could possibly emerge (an aspect related to the RPS of that individual).

For example, it may be claimed that a person should be at liberty to(or not to) whistle a tune, should be free to determine that aspect(i.e. whether or not he or she whistles) of the overall social state. This corresponds to what Gaertner et al. call the 'intuitive conception' of liberty. If a person has such an aspect choice, then he or she is given inter alia the power to choose between various pairs of different social states, given the respective choices of the other aspects (done by other persons or the nature).

But the converse is not the case, i.e. a person having the minimal liberty to make a choice over two social states does not, obviously, have the general right of choice over an aspect no matter what the others do.

(3) Existence of a right as distinguished from its value:

A right gives a person a certain opportunity. The existence of a right has to be distinguished from the value of that opportunity. A right may be of no use at all to a person for various reasons. For example, there may be no opportunity to use it; there may be no gain in using

it. But the absence of use would not obviously compromise the existence of that right.

(4) Private sphere distinguished from personal agency:

Sometimes the crucial agency of decision may not rest on the individual himself. For example, one's liberty of not to have smoke blown on to your face by a no-nonsense smoker , or your liberty to sleep peacefully at night without having to listen to loud music coming from next door, depends greatly on the actions of others. But these are indeed matters of one's own personal life and liberty. This type of case may be described as 'invasive actions', in which other people's actions invade one's private sphere.

The impossibility result involving liberty and the Pareto principle uses a condition of minimal liberty which requires that atleast two persons have the right to be decisive over one pair of social states each. The so-called 'Liberal Paradox' establishes that no Social Decision Function can satisfy both minimal liberty and the Pareto principle. There are various interpretations for the social preference, P and individual preference, P_i. But their interpretation in terms of choice is most in line with the usual conceptions of liberty.

Two interpretations of the social preference xPy are:

Social Choice: y should not be the outcome in the choice over any set that contains x.

Social Judgement: x is socially judged better than y.

Similarly, x P_ky can be interpreted as:

Individual Choice: Person k does not choose y from any set that contains x.

Individual Desire: Person k desires that x be chosen rather than y.

Individual Happiness: Person k is happier if x is chosen rather than y.

The focus on choice interpretation has the following limitations:

- A person's actual choices may be influenced by circumstances that could be appropriate to consider in judging whether a person really did have liberty in a significant sense. Social influences may induce a person not to choose in the way he or she would really like.

  One way of dealing with cases of these types, in which choice inhibition plays a major part, is to relate the outcome to what the person wants or desires, rather than concentrating only on what was or was not chosen.

- We really cannot neglect the question of 'invasive actions' in dealing with liberty in private matters. If one does not want smoke blown onto one's face, it is not just a matter of one's choice of action, but also that of others—the human chimneys in particular. The desire interpretation which relates

one's liberty to what one desires has a clear advantage in this case.

Sen on Gibbard's Extension

Gibbard's escalated condition demands that whenever two social states differ only in an aspect directly relevant for any one given person, that person should be decisive over that pair in terms of social choice. Gibbard showed that this more demanding condition of liberty is inconsistent with itself. The Gibbard Paradox has led to a large and important literature.

How substantial is the Gibbard extension in changing the characterization of the minimal libety in the ML formulation since two of the three problems discussed by Gaertner et al. apply to Gibbard's extension, but not to the original ML formulation itself?

Is it natural to agree to the Gibbard extension, given the acceptance of ML? Sen belives that it is not so. This is because in the choice framework ML is much weaker than the intuitive conception whereas the Gibbard condition is, in some ways, much more demanding

In Sen's ML framework, each person is guaranteed to be decisive over only one pair each, so that neither a person's ability to match the other's shirt nor his ability to differentiate from the other, can be guaranteed. In this case, obviously the Gibbard cycle cannot arise.

Since two of the three problems in

Gaertner *et al.*'s critique of Sen apply not to ML but to the Gibbard extension, they cannot be accepted as sufficient critiques of ML. But Sen agrees with Gaertner et al that it would be most unreasonable to demand that liberty should have those implications; but these implications are not part of the ML conception itself.

The next question that Sen considers is: Is the game form structure robust enough for an adequate specification of liberty?

The main source of complexity in individual choices lies in interdependence of individual rights. Social choice formulations can deal with such interdependence easily enough since the rights are characterised with explicit reference to outcomes or combination of strategies. To have similar sensitivity, game form formulations of a person's rights must also draw on the information regarding the actions of others, and it would not do to go instead in the direction of what Gaertner *et al.* call the intuitive conception.

Intuitive Conception and Minimal Liberty

How does minimal liberty relate to what may be called the common intuitive conception of liberty?

The particular intuitive conception that Gaertner *et al.*(1992) identify is as follows:"Under our intuitive conception of the right to choose one's own shirt, the individual enjoys the power to determine a particular aspect or feature (i.e. the colour of his own shirt) of the social alternative, and when he makes his choice w.r.t this particular aspect, his choice imposes restriction on the final social outcome, insofar as, in the final social outcome, that particular aspect must be exactly as he chose it to be".

Note first that this conception is concerned only with the choice view. Note also that it deals with a case in which the problems of invasive actions does not arise, and there is an assumed congruence of private sphere and personal agency. In contrast, the social choice framework permits variations in interpretations to cover various types of problems

Under the 'intuitive conception' the individual enjoys the power to determine a particular aspect or feature of the social alternative, but that gives the person the right to determine the choice over atleast one pair of social alternatives. Thus, within the limited context of the intuitive conception, anyone endorsing that conception must also endorse ML but not vice-versa.

Gaertner *et al.* do not however see the matter in this way, they believe that the two conceptions are different and fixing one aspect in the so-called intuitive conception is much more powerful than what ML depends on. They believe that the intuitive conception is not only different from but can also run counter to the conception of liberty underlying ML. They identify three problems (A, B

and C) that affect ML and its extensions. As Sen would argue, in the critics' example of the choice of colour of the shirt, (b,w) appears as the social outcome despite it being the worst outcome for individual 1 out of {(w,w),(b,w)}. But what does it show? Not that person 1 did not have the liberty to exclude the choice of (b,w) in favour of (w,w), but only that he chose not to use his right. By choosing w person 1 could have guaranteed the exclusion of (b,w) in favour of (w,w), but he chose differently in trying to ensure the avoidance of a worse outcome (w,b) in another sphere.

The fact that person 1 does not exercise a right, even the fact that this right may not be terribly valuable to him, given his preference ordering, does not affect the fact that he had the right and the power to knock out the choice of (b,w) in favour of (w,w).

So, Sen believes that what Gaertner *et al.* call the intuitive conception of liberty is a choice based conception that is limited in many ways particularly by its neglect of invasive actions and choice inhibition. But it does entail the social choice ML formulation in the choice interpretation, although it is not entailed by it. The desire conception of ML, being on a different plane of reference, may not be entailed by the choice based intuitive conception, but when the two clash, there is a substantive issue to be determined as to whether the choice view or the desire view would be more faithful to our traditional concept of liberty.

The critics (Gaertner et al.) do not agree with Sen regarding the similarity of the views as the difference between the Sen and Gibbard formulations is very large and the intuitive notion of liberty lies somewhere in between.

Refutation of Sen's Assertion

Consider the example where the individuals have to choose the colour of their shirt, the preference profile being:

1: $(w,w)P_1(b,b)P_1(b,w)P_1(w,b)$

2: $(b,w)P_2(w,b)P_2(w,w)P_2(b,b)$

The intuition of 1's liberty to choose his shirt (following game forms) simply refers to 1's freedom of choosing b or w. This freedom in turn means that 1 has the power to exclude the realisation of both (w,w) and (w,b) (respectively both (b,b) and (b,w)) by choosing b (respectively w) for himself.

Given 1's preferences, Gibbard's rights empower 1 to exclude the realisation of both (b,w) and (w,b).

Therefore, the two views are really non-comparable in terms of their relative strengths and it is not true that the Gibbard's view of rights demands more than the intuitive conception of rights.

Regarding the notion that the intuitive

view demands more than ML, suppose 1 has the Sen right for (w,w) against (b,w). Suppose 1's preferences are given by $(w,w)P_1(b,w)P_1(b,b)P_1(w,b)$

For lack of knowledge of 2's choice, 1 may choose b in accordance with the maximin principle. Then the social alternatives which will be ruled out are (w,w) and (w,b). The social outcome depends on 2's choice when he exercises his right.

If 2 has the preferences $(b,w)P_2(w,b)P_2(w,w)P_2(b,b)$ and if also follows the maximin principle then he will choose w and therefore (b,w) will emerge as the social outcome which ML rules out.

Thus, the intuitive view does not imply ML. Note that if 1 had full knowledge of 2's choice or if 1 had a dominant strategy, the intuitive notion of liberty would imply ML. But in the context of the right to choose the colour of one's own shirt, the uncertainty[5] that an individual faces regarding the other's colour choice is an integral part of the problem. Also the existence of a dominant strategy for a player is very much an exception rather than a rule.

In the case of uncertainty and absence of a dominant strategy, the above problem arises. Sen substantiates his stand on such occasions by saying that 1 is not exercising his right.

The condition ML is ethically unacceptable to the critics. According to them, if at the time of choosing strategy b, 1 does not know whether 2 will choose b or w

nevertheless, once 1 and 2 choose the strategies b and w respectively, Sen's interpretation would be that 1 has chosen (b,w) over (b,b), but normally we never speak of an individual choosing over social states, we refer to an individual choosing an aspect of the social state .

They believe that it would seem much easier if ML is given up and the essence of the right involved in Sen's example lies in the individual's freedom to choose any of the options available to him, and that, regardless of the final social outcome, there is no violation of the individual's right so long as he is free to choose his option.

Further the critics believe that under virtually every plausible definition of individual preferences, the condition ML can conflict with the intuitive notion of liberty if the individual concerned is faced with uncertainty regarding other individual's choice of strategies and if he does not have a dominant strategy.

Contracting as Solution to Impossibility Result

The impossibility of the Paretian liberal(i.e. Liberal Paradox) has led to a substantial literature on interpreting, resolving and extending the problem. One suggestion for resolution that has occurred to many is that the people involved may contract to bring about a Pareto improving solution, since that improves the position of all. In the infamous case of *Lady Chatterley's Lover*, Prude can promise to read that

book provided Lude promises not to read it, and this would take them to a Pareto-superior position.

Note that a Pareto-improving contract is always a possibility in any Pareto inoptimal situation. But we have to consider the question of credibility of such a contract and its compliance (i.e. how to make sure that Prude actually reads the book and not just pretends to, and how to guarantee that Lude is not turning pages on the sly).

Such enforcement would be necessary if people were to conform voluntarily to the agreement. If the individual preferences P_i is taken on its choice interpretation, then this possibility is not open (without changing the orderings), since the postulated orderings indicate they will choose otherwise.

It is not at all obvious why Prude and Lude must act inescapably to go for a peculiarly 'other-regarding' social contract by which

- Prude agrees to read a book he hates in order to refrain Lude from reading it; and

- Lude in turn agrees to forgo reading a book he would love to read in order to induce the reluctant Prude to read it instead.

If people attach some importance to minding their own business, then that odd contract need not infact materialize. The good liberal practice of reading what one likes and letting other's read what

they like can perhaps survive the alleged temptations of having this remarkable contract.

For some inexplicable reason, some authors seem to believe that the issue in question is whether rights are alienable and whether the persons involved would be allowed to have such a contract. It is hard to think that what has been called alienability can, in general, be in dispute here. There is no reason why rights of this kind should not be in general taken to be open to contracting and exchange through mutual agreement.

There can be little doubt that people do not in general need anyone else's permission to have such a contract. But they do need a reason. To cite as a reason the fact that such a contract would be the only way of getting—and sustaining—a Pareto optimal outcome is to beg the question, since the motivation for discussing the impossibility result is precisely to question and assess the social merits of Pareto optimality.

The real issue concerns the adequacy of the reasons (a) for having such a contract; and (b) for sticking to it. Of course, no non-sense maximisation of pleasure or desire fulfilment could provide some reason for seeking or accepting such a contract.

If the persons are free to have or not to have such a contract, the dilemma of the Paretian Liberal remains. The Pareto

libertarian conflict resurfaces here in the dilemma of personal behaviour.

Sen on Liberal Paradox Resolution

Sen believes that the paradox arises due to the stringent nature of WP i.e. whatever is said about one pair gets passed on to all possible pairs of alternatives, i.e. if an individual is decisive over a single pair he gets semi rights[6] to be decisive over all pairs of alternatives including those that lie in other individual's RPS

So he suggests that WP should be weakened in order to resolve the conflict with the libertarian condition. But this solution does not prove to be adequate in resolving the conflict.

VII. CONCLUDING POINTS

We have considered two important aspects involving the inclusion of individual rights in social decision making:

- How to formulate individual rights?
- How to resolve the conflict arising between WP and liberalism?

As regards the first question, we looked at three formulations: Sen's social-choice formulation, Gibbard's formulation, and the game form approach (or the intuitive notion approach given by Gaertner et al. and Nozick).

The first two formulations look at individual choices over social states whereas the game form emphasises individual choice over particular aspects of the social alternatives. Following are the main points regarding the three formulations:

(i) All the approaches are quite different from each other though they might have some similarities. One can always think of a situation where the approaches are in conflict with each other (i.e. one does not imply the other).

(ii) The demands of liberty are diverse and the problems of realisation or violation of liberty can arise in quite different contexts (some of the variations can relate to the basis of the demands of liberty and to the field of application). These diverse cases have intrinsic relevance in different contexts.

Different formulations are suited for different situations. For example, the social choice approach of Sen is helpful in the case of independence of individual rights and in the case of invasive actions. The game form approach can be usefully employed to analyse different distributions of rights and the possible equilibria.

The second important question that we considered is as to how to resolve the Liberal Paradox. Our analysis shows that the paradox would persist under virtually every plausible concept of individual rights that one can think of.

The claim that Pareto improving contracts can help resolve the conflict does not have a sound basis since the

dilemma remains so long as the individuals are free to have or not to have such a contract.

The main reason behind the emergence of Liberal Paradox is the nosey nature of individual preferences, i.e until individuals would be concerned with others' personal affairs we would have liberal paradox. The main problem here is to distinguish an individual's personal sphere in which no one should be allowed to interfere.

Notes

1. An ordering is a binary weak preference relation which satisfies the following three conditions:- Reflexivity: $\forall x \in X$, xRx, i.e. an alternative is at least as good as itself;-Completeness: $\forall x,y \in X$, either xRy or yRx or both, i.e. all the alternatives can be ranked;- Transitivity: $\forall x,y,z \in X$, if xRy and yRz then xRz.

2. Acyclity means that there is absence of preference cycles, i.e. $\forall x_1, \_ , x_n$ if $x_1 P x_2$, $x_2 P x_3$, $\_ \_ \_ \_ x_{n.}$, $P x_n$ then $\sim x_n P x_1$.

3. Maximin principle is a decision rule in the theory of games where a player with a number of optional strategies to choose from, first considers the minimum payoffs that could be gained from each,

depending on the reactions of his opponent. And then the player should choose that strategy which corresponds to the maximum of all the minimum payoffs that are possible to him.

4. A course of action that is best pursued whatever it is that other agents choose to do.

5. A state where the number of possible outcomes exceeds the number of actual outcomes and when no probabilities can be attached to each possible outcome.

6. An individual k is said to have semi-rights over an ordered pair (x,y) if for some $A \in \chi$ such that $x,y \in A$ and for all profiles(R) belonging to $\Re^n$, $x P_i y \rightarrow \{x\}=C(A,R)$.

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